Objectives:

- Define definite integrals.
- Find areas under curves using definite integrals.

Definitions: If f is a function defined for $a \leq x \leq b$, we divided the interval $[a, b]$ into n subintervals of equal width

$$\Delta x =$$

We let $x_0 = a$, x_1, …, $x_n = b$ be the endpoints of these subintervals and we let x_1^*, $x_2^*, \ldots, x_n^*$ be any ____________________ in these subintervals, so x_i^* is in the ith subinterval $[x_{i-1}, x_i]$. Then the **definite integral of f from a to b** is

provided the limit exists. If the limit does exist, we say that f is __________________________.

Terminology: Let's break down the notation $\int_a^b f(x)\,dx$.

- The symbol $\int$ is called an ___________________
- $f(x)$ is the ________________
- a and b are the _________________________
- a is the ______________________________ and b is the ______________________________
- We call computing an integral _________________

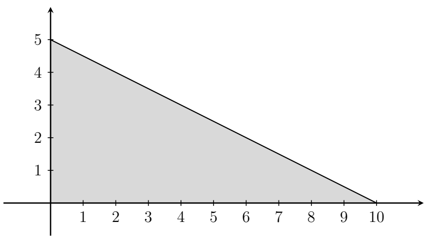
Some intuition: The definite integral is computing ____ but we consider any area above the x-axis is _______________ and any area underneath the x-axis is _______________.

But wait! Our definition shows that the definite integral is also ________________________________

Some useful things:

- The sum of the integers from 1 to n: $\sum_{i=1}^{n} i =$
- The sum of the squares of integers from 1 to n: $\sum_{i=1}^{n} i^2 =$
- The sum of the cubes of integers from 1 to n: $\sum_{i=1}^{n} i^3 =$

Example 1 Write down a definite integral that gives the area of the shaded region.



Example 2 Evaluate $\int_0^3 12 - 6t \, dt$ by drawing a the region and computing the area.

Example 3 Evaluate $\int_0^2 \sqrt{4 - x^2} \, dx$ by drawing a the region and computing the area.

Example 4 A table of values of $f(x)$ is given below. Estimate $\int_0^{12} f(x) \, dx$ using Riemann sums.

x	0	3	6	9	12
$f(x)$	32	22	15	11	9

Example 5 Calculate $\int_0^2 x^3 \, dx$ exactly using a limit of Riemann sums.

Theorem If $f(x)$ is ________________, or if $f(x)$ has only a finite number of jump discontinuities, then f is ________________, i.e., the definite integral ________________ exists.

Things to note: We have assumed that $a < b$ for defining $\int_a^b f(x)\ dx$, but the Riemann sum will allow $a > b$. If $a > b$, then Δx used to be $\dfrac{b-a}{n}$ and is now ____________. So we have

$$\int_b^a f(x)\ dx =$$

What if $a = b$? Then $\Delta x =$ ____________ so

$$\int_a^a f(x)\ dx$$

Properties of Definite Integrals: Let $f(x)$ and $g(x)$ be continuous functions and c some constant number.

1. $\int_a^b c\ dx =$
2. $\int_a^b \big[f(x) + g(x)\big]\ dx =$
3. $\int_a^b cf(x)\ dx =$
4. $\int_a^b \big[f(x) - g(x)\big]\ dx =$
5. $\int_a^c f(x)\ dx + \int_c^b f(x)\ dx =$

Example 6 Evaluate $\int_0^2 (4 + 5x^3)\ dx$.